where f =force per sq. cm tending to displace one layer with respect to another

- $\lambda$  = distance between equilibrium positions in the direction of flow
- $\lambda_2$  = distance between adjacent molecules in the direction of flow
- $\lambda_3$  = distance between molecules in the plane of flow and normal to the direction of flow.

The area of a molecule in the plane of flow will be  $\lambda_2 \lambda_3$ , the force acting on the molecule will then be  $f\lambda_2\lambda_3$ , and if the force acts through a distance  $\lambda/2$ , the energy of activation will be raised or lowered by  $\frac{1}{2}f\lambda_2\lambda_3\lambda$ . Fig. 2 shows these energy relations diagrammatically.

The viscosity is given hydrodynamically by

$$\eta = \frac{f\lambda_1}{\Delta V} = \frac{f\lambda_1}{\lambda(k_f - k_b)}$$

where  $\lambda_1 =$  perpendicular distance between adjacent layers of molecules,

> $\Delta V =$  difference in velocity of two layers a distance  $\lambda_1$  apart

$$=\lambda(k_f-k_b),$$

so that

$$\eta = \frac{f\lambda_1}{\lambda k_1 (e^{f\lambda_2 \lambda_3 \lambda/2kT} - e^{-f\lambda_2 \lambda_3 \lambda/2kT})}$$
$$= \frac{f\lambda_1}{\lambda k_1^2 \sinh f\lambda_2 \lambda_3 \lambda/2kT}.$$
(2)

The expansion of  $\sinh x$  is  $\sinh x = x + 1/6x^3$  $+1/120x^5+\cdots$ , so that for ordinary viscous flow, where  $f\lambda_2\lambda_3\lambda/2\ll kT$ ,

$$\sinh f\lambda_2\lambda_3\lambda/2kT \cong f\lambda_2\lambda_3\lambda/2kT$$

and we have

$$\eta = \frac{\lambda_1 h}{\lambda^2 \lambda_0 \lambda_2} \frac{F_n}{F^*} e^{\Delta E_a/kT}.$$
 (3)

Eq. (2) may be considered the fundamental equation of flow, and also Eq. (3) with the limitation mentioned.

Now if we assume that a molecule flows one intermolecular distance in each elementary process, i.e., that  $\lambda = \lambda_2$ , and for an equant molecule  $\lambda_1 = \lambda_2 = \lambda_3$ 

$$\lambda_1/\lambda^2\lambda_2\lambda_3 = N/V$$

where N = Avogadro's number

V = molar volume.

If we assume that the degree of freedom corresponding to flow is a translational one, and that other degrees of freedom are the same for the initial and activated states:

$$\frac{F_n}{F_a^*} = \frac{((2\pi mkT)^{\frac{1}{2}}/h^3) V_f F_{\rm rot} F_{\rm vib}}{(2\pi mkT/h^2) V_f^{\frac{1}{2}} F_{\rm rot} F_{\rm vib}} = \frac{(2\pi mkT)^{\frac{1}{2}}}{h} V_f^{\frac{1}{2}}$$

where  $V_f$  = the free volume per molecule. Eyring and Hirschfelder<sup>4</sup> have formulated the free volume

$$V_{f}^{\dagger} = \frac{bRT}{V^{\dagger}N^{\dagger}(p+a/V^{2})} \text{ per molecule} \qquad \eta =$$

$$= \frac{bRTV^{\dagger}}{N^{\dagger}\Delta E_{\text{vap}}} \quad \text{if} \quad \frac{a}{V^{2}} = \frac{\Delta E_{\text{vap}}}{V} \gg p, \qquad =$$

where  $\Delta E_{\rm vap}$  = energy of vaporization per mole  $= \Delta H_{\rm vap} - \Delta(\rho v),$ 

> b=2 for simple cubic packing and has not very different values for other types of packing.



FIG. 2. Energy profile of the path of a flowing molecule.

As stated before the activation energy takes the form of providing a hole for the molecule to flow into, but this may not necessarily have to be a hole the full size of a molecule. A consideration of the attractive and repulsive forces involved in vaporization shows that for any liquid the energy

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ties of Eq.

$$\eta = -$$

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